AP Calculus BC

L'Hopital's Rule

1) $\lim_{x\to 0} \frac{x}{\tan x} = \frac{0}{0}$

$\overset{L'H}{=} \lim_{x\to 0} \frac{1}{\sec^2 x} = 1$

2) $\lim_{x\to 1} \frac{1-1/x}{1-1/x^2} = \frac{0}{0}$

$\overset{L'H}{=} \lim_{x\to 1} \frac{1/x^2}{2/x^3} = \frac{1}{2}$

3) $\lim_{x\to\infty} \frac{\log_2 x}{\log_3 x} = \frac{\infty}{\infty}$

$\overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{1}{x\ln 2}}{\frac{1}{x\ln 3}}$

$\lim_{x\to\infty} \frac{x\ln 3}{x\ln 2} = \boxed{\frac{\ln 3}{\ln 2}}$

4) $\lim_{h\to 0} \frac{8\left(\frac{1}{2}+h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h} = \frac{0}{0}$

$\overset{L'H}{=} \lim_{h\to 0} \frac{64\left(\frac{1}{2}+h\right)^7 \cdot (1)}{1} = 64\left(\frac{1}{2}\right)^7 = \boxed{\frac{1}{2}}$

5) $\lim_{x\to 0} \frac{e^{2x}-1}{\tan x} = \frac{0}{0}$

$\overset{L'H}{=} \lim_{x\to 0} \frac{2e^{2x}}{\sec^2 x} = \boxed{2}$

6) $\lim_{m\to 0} \frac{1}{m} \ln\left(\frac{2+m}{2}\right) = \infty \cdot 0$

$\lim_{m\to 0} \frac{\ln\left(\frac{2+m}{2}\right)}{m} = \frac{0}{0}$

$\overset{L'H}{=} \lim_{m\to 0} \frac{\frac{1}{2+m}}{1} = \boxed{\frac{1}{2}}$

7) $\lim_{n\to\infty} \frac{4n^2}{10000n + n^2} = \boxed{4}$

8) $\lim_{\theta\to 0} \frac{\arctan\theta}{2\theta} = \frac{0}{0}$

$\overset{L'H}{=} \lim_{\theta\to 0} \frac{\frac{1}{(1+\theta^2)2}}{} = \boxed{\frac{1}{2}}$

9) $\lim_{v\to\infty} \frac{v^2}{e^v} = \boxed{\infty}$

10) $\lim_{x\to\pi^+} \frac{2x-2\pi}{\sin(x-\pi)} = \frac{0}{0}$

$\overset{L'H}{=} \lim_{x\to\pi^+} \frac{2}{\cos(x-\pi)} = \boxed{2}$

11) $\lim_{x\to\pi} \frac{1+\sec x}{x^2-\pi^2} = \frac{0}{0}$

$\overset{L'H}{=} \lim_{x\to\pi} \frac{\sec x \tan x}{2x} = 0$

12) $\lim_{x\to 0} \frac{\sin x}{\ln(2e^x-1)} = \frac{0}{0}$

$\overset{L'H}{=} \lim_{x\to 0} \frac{\cos x}{\frac{2e^x}{(2e^x-1)}}$

$\lim_{x\to 0} \frac{2e^x-1}{2e^x \cos x} = \boxed{\frac{1}{2}}$

13) $\lim_{x\to\infty} \frac{2x+3e^{-5x}}{g(x)} = \frac{\infty}{\infty}$

$\overset{L'H}{=} \lim_{x\to\infty} \frac{2-15e^{-5x}}{\frac{1}{x}+4\cos\left(\frac{5}{x}\right)} = \frac{2}{4} = \boxed{\frac{1}{2}}$

14) $\lim_{x\to 3} \frac{xf(x)-6}{g(x)+1} = \frac{3f(3)-6}{g(x)+1}$

$= \frac{0}{0}$

$\overset{L'H}{=} \lim_{x\to 3} \frac{xf'(x)+f(x)}{g'(x)}$

$= \frac{3f'(3)+f(3)}{g'(3)}$

$= \boxed{-\frac{5}{2}}$